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CALCULATION OF THE EFFECT OF CLOSED AIR CIRCULATION ON THE EQUILIBRIUM DISTRIBUTION OF OZONE IN THE EARTH'S ATMOSPHERE

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SUMMARY

This paper is devoted to the calculation of ozone density distribution in the atmosphere in the presence of circulation and as a function of latitude of the place.

* * *

It has been experimentally established that the ozone content in the atmosphere and its distribution in height depend essentially on vertical and horizontal air currents in the atmosphere [1]. Accordingly, the distribution of ozone may serve as an important indicator of atmospheric processes and even of the synoptic positions. This imparts the investigation of ozone and to its variations in the terrestrial globe scale a direct practical interest.

Theoretically the effect of purely vertical currents on the distribution of ozone is treated in the work [2].

In the present work we consider the influence on ozone distribution of a concrete scheme of general atmosphere circulation, including both, the horizontal and vertical branches of the flow.

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* [O RASCHETE VLIYANIYA ZAMKNUTOY VOZDUSHNOY TSIRKULYATSII NA RAVNOVESNOYE RASPREDELENIYE OZONA V ZEMNOY ATMOSFERE]

We shall consider a closed air current [3], of which the vertical and horizontal velocity components, V_r and V_φ, taking into account the continuity equation for the exponentially decreasing with height air density, may be represented in the form

$$V_r = V_0 \exp\{(\mu - 2(R)(r - R_3)\} \cos b(r - R) \sin aR(\varphi - \varphi_0) / \sin\varphi, \qquad (1)$$
$$V_\varphi = -bV_0 \exp\{(\mu - 2/R)(r - R_3)\} \sin b(r - R) \cos aR(\varphi - \varphi_0) / a \sin\varphi.$$

Here r is the radius-vector, directed from the center of the Earth; R is the distance from the latter to the height of the center of the region of flow; R_3 is the Earth's radius, φ being the complement to the latitude; $\varphi_0 = \pi/4$ is the angle of the region's center; V_0 is a parameter characterizing the magnitude of the velocity; $\mu = 0.125\ km^{-1}$ is the reciprocal to the "height of uniform atmosphere", characterizing the rate of air density decrease with height [4]; a and b define the dimensions of the flow region in such a fashion, that there be $V_\varphi = 0$ at the northern and southern boundaries of the region, and $V_r = 0$ at the upper and lower, that is

$$a = \pi / 2R|\Delta\varphi_{max}|,$$
$$b = \pi / 2|\Delta r_{max}|,$$

where $\Delta\varphi_{max}$ and Δr_{max} are the maximum deflections φ and r from the cor sponding coordinates of the region's center.

The current with velocity components (1) will have the form indicated in Fig. 1 by arrows.

The continuity equation for ozone density, with the right-hand part depending on diffusion and photochemical processes in the atmosphere, determining the formation and destruction of ozone,

$$\partial\rho / \partial t + \operatorname{div}(\rho \mathbf{V}) = \alpha(\rho_0 - \rho) + D\nabla^2\rho, \qquad (2)$$

is resolved in this region. Here $\mathbf{V}$ is the flow velocity vector; α is a quantity reciprocal to time of ozone semi-regeneration ; D is the diffusion coefficient; ρ_0 is the equilibrium ozone density at $\mathbf{V} = 0$, conditioned only by photochemical processes. It is admitted that

$$\rho_0 = 17(H - 10)^3 e^{-0{,}25H} \begin{cases} (1 + \sin 2\varphi) & \varphi < \pi/4, \\ [1 + (2 - \sin 2\varphi)] & \varphi > \pi/4, \end{cases} \qquad (3)$$

where H is the height above ground.

The distribution of ozone is searched for in a settled air current, in which $\partial\rho / \partial t = 0$. In the first approximation, considered here, the effect of diffusion is not taken into account, that is $D = 0$.

Under these assumptions the equation (2) will have the form

$$\frac{\partial(\rho V_r)}{\partial r} + \frac{1}{R}\frac{\partial(\rho V_\varphi)}{\partial\varphi} + \frac{2\rho V_r}{R} + \frac{\rho V_\varphi \operatorname{ctg}\varphi}{R} = \alpha(\rho_0 - \rho). \qquad (4)$$

Here div (ρV) is expressed in spherical coordinates with two variables and not in polar ; it is done so with the view of taking into account the meridian convergence to the pole in the simplest possible manner. The unique solution of the equation (4) will be arrived at if we start from the condition of ozone density continuity; the latter is determined by various methods in the four sub-regions of our region. We shall dwell upon this at further length in the following.

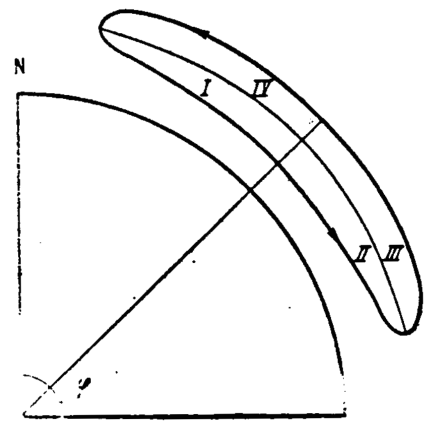

Fig. 1

Upon substitution in it of V_r and V_φ according to the expressions (1), the equation (4) will be transformed into

$$\frac{\partial\rho}{\partial r}\cos b(r-R) - \frac{\partial\rho}{\partial\varphi}\frac{b}{aR}\operatorname{ctg} aR(\varphi-\varphi_0)\sin b(r-R) =$$

$$= -\rho\left[\mu\cos b(r-R) + \frac{\alpha}{V_0}\Phi(r,\varphi)\right] + \frac{\alpha\rho_0}{V_0}\Phi(r,\varphi), \qquad (5)$$

where

$$\Phi(r,\ \varphi) = \exp\left(\frac{2}{R} - \mu\right)(r - R_3)\frac{\sin\varphi}{\sin aR(\varphi-\varphi_0)}.$$

This linear nonuniform equation is resolved in partial derivatives of first order by a standard method, with the aid of integration of an auxiliary system of ordinary differential equations [6]

$$\frac{dr}{\cos b(r-R)} = -\frac{d\varphi}{b\operatorname{ctg} aR(\varphi-\varphi_0)\sin b(r-R)/aR} =$$

$$= \frac{d\rho}{-\rho[\mu\cos b(r-R) + \alpha\Phi(r,\varphi)/V_0] + \alpha\rho_0\Phi(r,\varphi)/V_0}. \qquad (6)$$

This system's first integral, which is easy to obtain by integrating the first equation of the system, has the form

$$\cos b(r-R)\cos aR(\varphi-\varphi_0)=C. \tag{7}$$

After that the solution of the equation is reduced to the integration of the ordinary differential equation of first order

$$\frac{d\rho}{dr}+\left[\mu+\frac{\alpha}{V_0}\frac{\Phi(r,\varphi)}{\cos b(r-R)}\right]\rho=\frac{\alpha\rho_0}{V_0}\frac{\Phi(r,\varphi)}{\cos b(r-R)}, \tag{8}$$

where φ is expressed through r with the aid of (7); this equation may be written more briefly in the form

$$d\rho/dr+p(r,c)\rho=f(r,c). \tag{8a}$$

The dependence of ozone distribution on photochemical processes is manifest strongest of all at heights from 20 to 30 km; it is thus important to assort a function reflecting the true course α precisely in that interval. On the basis of experimental data and calculations, A.V. Kondrat'yeva has chosen $\alpha=\alpha_0 e^{0,5H}=6\cdot10^{-13}e^{0,5H}$ sec^{-1} *.

The region of flow is taken in height above the Earth's surface from 2.5 to 37.5 km (the center is situated at 20 km height); by latitude it extends from the pole to equator (center: $\varphi=\pi/4$). This is obtained by postulating b = 0.09, aR = 2; then b / a $\approx$ 287.

The solution of the equation (8a) has the form

$$\rho(r,c)=C\exp\left\{-\int p(r,c)\,dr\right\}+\exp\left\{-\int p(r,c)\,dr\right\}\times$$
$$\times\int f(r,c)\exp\left\{\int p(r,c)\,dr\right\}dr. \tag{9}$$

At $\varphi=\pi/4$ the quantities p (r, c) and f (r, c) pass to infinity; however, it may be assumed in this case, as should be expected, that the solution will be finite. Physically this means that the variation of ozone density in height is not manifest in the solution on the line for there are no vertical air currents on that line.

In order to conduct integration in the right-hand part of the equality (9), one must represent the trigonometric functions in the expressions for p (r, c) and f (r, c) approximately in the form of polynomials from x = bh (h = H − 2.5 km); these polynomials will have, at the same time,

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* Expression for α derived on the basis of calculations in the thesis by Kondrat'yeva, published in 1962 by the Moscow State University.

a different form at $\varphi < \pi/4$ and $\varphi > \pi/4$. At $\varphi < \pi/4$ all the functions will have the index 1, and at $\varphi > \pi/4$ — the index 2.

By the strength of solution's continuity, its "joining" will be effected at $\varphi = \pi/4$, that is, it will be assumed that $\rho_1(r, \pi/4) = \rho_2(r, \pi/4)$.

It is admitted approximately, that

$$\frac{\sin\varphi}{\cos b(r-R)\sin aR(\varphi-\varphi_0)} \approx$$

$$\approx \begin{cases} [(0{,}05 - 64c^5)x^4 - (0{,}314 - 401{,}92c^5)x^3 + \\ + (0{,}84 - 954{,}56c^5)x^2 - (1{,}088 + 1015{,}84c^5)x + \\ + (0{,}75 - 412{,}584c^5)] \quad \text{at} \quad \varphi < \pi/4, \\ [(0{,}125 + 64c^5)x^4 - (0{,}785 + 401{,}92c^5)x^3 + \\ + (2{,}099 + 954{,}56c^5)x^2 - (2{,}72 + 1015{,}84c^5)x + \\ + (1{,}876 + 412{,}584c^5)] \quad \text{at} \quad \varphi > \pi/4. \end{cases} \tag{10}$$

The expressions (10) decrease our region of flow, since we may utilize them only in specific interval of H and φ variation. We shall conduct the computations for a region, bounded in height by 10 and 30 km, and by latitudes 20° and 70°.

The expressions (10) poorly approximate the initial function at near $\pi/4$. That is why in the band $42.5^\circ < \varphi < 47.5$ the calculations will not take place and it will be approximately admitted that $\rho_1(x, 42{,}5^\circ) = \rho_2(x, 47{,}5^\circ)$. — This is admissible, since in this region the vertical air currents are very small, and basically they do determine the ozone density gradient (as will be seen further).

Utilizing this condition of continuity, we may write the solution independently from the "joining" at $\varphi = 42.5 - 47.5^\circ$:

$$\rho_{1,2}(x,\varphi) = \dot{\rho}_{1,2}(\varphi)\frac{P_{1,2}(x,c)\,|_{x=x_0}}{P_{1,2}(x,c)} - \frac{F_{1,2}(x,c)\,|_{x=x_0}}{P_{1,2}(x,c)} + \frac{F_{1,2}(x,c)}{P_{1,2}(x,c)}, \tag{11}$$

where

$$P_{i,j}(x,c) = \exp\left\{\int p_{ij}(x,c)\,dx\right\},$$

$$F_{i,j}(x,c) = \int f_{ij}(x,c)\exp\left\{\int p_{ij}(x,c)\,dx\right\}dx,$$

while the solution, dependent upon the "joining" will be :

../..

$$
\begin{aligned}
\rho_{1,2}(x,\varphi) = \rho_{1,2}^{*}(\varphi)\, & \frac{P_{2,1}(x,c)\,|_{x=x_0}\, P_{1,2}(x,c)\,|_{\varphi=\varphi_0}}{P_{2,1}(x,c)\,|_{\varphi=\varphi_0}\, P_{1,2}(x,c)} - \\
- \frac{P_{1,2}(x,c)\,|_{\varphi=\varphi_0}\, F_{2,1}(x,c)\,|_{x=x_0}}{P_{2,1}(x,c)\,|_{\varphi=\varphi_0}\, P_{1,2}(x,c)} & + \frac{P_{1,2}(x,c)\,|_{\varphi=\varphi_0}\, F_{2,1}(x,c)\,|_{\varphi=\varphi_0}}{P_{2,1}(x,c)\,|_{\varphi=\varphi_0}\, P_{1,2}(x,c)} - \\
& - \frac{F_{1,2}(x,c)\,|_{\varphi=\varphi_0}}{P_{1,2}(x,c)} + \frac{F_{1,2}(x,c)}{P_{1,2}(x,c)},
\end{aligned}
\tag{12}
$$

where, upon integration, c is substituted by its expression (7). $\varphi_0 = 42.5$ or 47.5°, x_0 corresponds to the value of x at $H = 20$ km, $\rho^*(\varphi)$ is the density of ozone at $H = 20$ km.

The functions $p_1(x,c)$, $p_2(x,c)$, $f_1(x,c)$, $f_2(x,c)$ represent the products of the polynomial from x by the exponent. The expressions of the type $\int f_i(x,c)\exp\{\int p_i(x,c)\,dx\}\,dx$ $(i = 1, 2)$ are integrated, expanding the exponent in series and limiting ourselves to two terms.

The ozone density ρ_1 in the region I (Fig. 1) is determined by formula (11), the density of ozone in the region II (ρ_2) is determined by the formula (12), the density ρ_2 in the region III — by (11) and the density ρ_1 in the region IV — by formula (12). At the level $x = x_0$, i. e. at $H = 20$ km, we may compute the ozone density at $\varphi < \varphi_0$ simultaneously for the regions I, IV and at $\varphi > \varphi_0$ — for the regions II, III. By the strength of the continuity of the function $\rho(x,\varphi)$, the values of this functions at $x = x_0$ are equated at each point, and, according to the obtained sequence of points we may assort $\rho^*(\varphi)$ — the density of ozome at $H = 20$ km. Therefore, by the strength of the closed state of current lines, the "boundary condition" is not pre-assigned, and it is obtained from the solution itself, having at $V_0 = 10^{-7}$ km/sec the form

$$
\begin{aligned}
\rho_1^*(\varphi) &= 215\left(1 + \exp\{-10/|\varphi - 45^\circ|\}\right),\\
\rho_2^*(\varphi) &= 215\left(1 - \exp\{-10/|\varphi - 45^\circ|\}\right),
\end{aligned}
$$

and at $V_0 = 2\cdot 10^{-7}$ km/sec (13)

$$
\begin{aligned}
\rho_1^*(\varphi) &= 215\left(1 + \exp\{-5/|\varphi - 45^\circ|\}\right),\\
\rho_2^*(\varphi) &= 215\left(1 - \exp\{-12{,}5/|\varphi - 45^\circ|\}\right).
\end{aligned}
$$

The ozone density was computed at every 5° latitude from 20 to 70° and at every 2 km height from 10 to 30 km. Above 30 km the calculation can no longer be conducted, as the limiting of exponent's expansion in series to two terms will be already insufficient. Our scheme assumes that above 37.5 km there generally are no currents, while commencing at about 35 km the vertical flows are very small, so that from that height the ozone distribution may be estimated to be near the photochemical. At 30 — 35 km heights we interpolate the approximate course of ozone density.

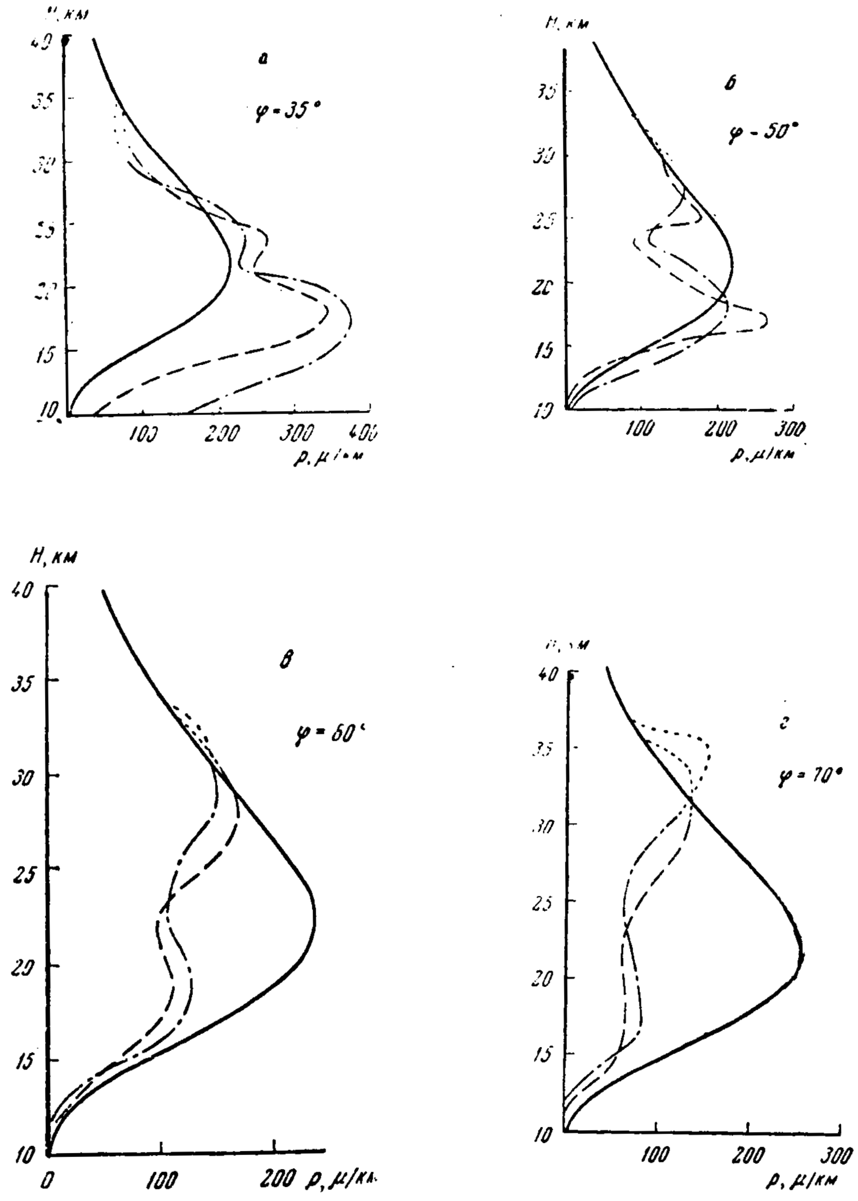

Fig. 2

The computations were effected for two different velocities V_0 determining the intensity of the whole circulation : 10^{-7} and $2 \cdot 10^{-7}$ km/s.

The value $V_o = 10^{-7}$ km/sec corresponds to the maximum vertical velocity ~ 0.4 cm/sec and to the minimum horizontal ~ 1.3 m/sec.

The examples obtained by formulas (11) and (12) of the results of distribution of equilibrium density of ozone in the presence of atmosphere circulation at $\varphi = 35, 50, 60, 70^o$ are plotted in Fig. 2.

The curves of ozone density are plotted by dashes for $V_o = 10^{-7}$ km/s and by stroke-dots — for $2 \cdot 10^{-7}$ km/ sec. The solid curve indicates the distribution of ρ_o at $V_o = 0$, according to the expression (3).

It may be seen from these curves, that the indicated scheme of atmos;here currents provides the distribution of ozone density coinciding in the whole with the experimental data. Thus, at high latitudes, where the downward currents are greatest, a sharp increase in the total ozone content is observed alongside with the lowering of the maximum and the increase in it of ozone concentration. As to the low latitudes, where ascending currents are prevalent, the total ozone content drops sharply, the maximum shifts upward, and the concentration of ozone in it diminishes. A dual maximum of ozone concentration is also frequently observed. For the given air current scheme, the upper maximum forms the ascending air current, and the lower one — the "residual" maximum, is conditioned by the maximum formed by descending air currents at $\varphi < \pi/4$. This maximum gradually dissipates on account of acsending currents and at lowermost latitudes it practically vanishes entirely.

It may also be seen from these curves, that the vertical currents exert a great influence on the distribution of ozone. The greatest variations of ozone content take place precisely where prevail the greatest vertical flows. The horizontal currents also influence the distribution of ozone, but to a substantially lesser degree.

One may notice also that there exists an interval of air current's vertical velocity, at which the variations of velocity are most manifest in the distribution of ozone. Beyond the interval, the velocity variations of a settled current are influencing the ozone distribution immaterially.

The most essential distinction between the results of calculation and the observations consists in a sharply overrated ozone content at high latitudes at comparatively low current velocities. This may be partially explained by the fact, that we consider a settled current, whereas in real atmospheric conditions it more or less approaches the settled state, partiy because of the distinction of our scheme from that of [3], which consists in that the last current, directed from south to north, spreads to the height of 70 — 80 km, while in our scheme the same amount of air is "accomodated" to 37.5 km height. That is why the "effective" air velocity in our scheme results greater. It is nevertheless possible, that the increased destruction of ozone in the lower layers should also be taken into account, that is, the aerosol oxidation etc..., which, so far, was disregarded here.

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*** THE END ***

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